November 2014 subject reports

MATHEMATICAL STUDIES

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 29	30 - 40	41 - 53	54 - 66	67 - 78	79 - 100

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

There was a marked improvement from the May 2014 session. It appeared as if many more teachers had followed the new criteria or had carefully read the May 2014 subject report.

There was a wide range of marks as usual. Most of the topics were statistical and were suitable for a Mathematical Studies SL project but there are always a few that should have been actively discouraged by the teachers. Some candidates had obviously worked hard on their project and enjoyed the process and this was obvious from the care that was taken to satisfy all the assessment criteria and, as a result, these candidates scored highly on all of the criteria. However, there were others that showed little, if any, commitment and produced a trivial or incomplete piece of work. Most schools did realize that the projects had to include two simple processes first before a further process was attempted and this lead to an increase in the marks for criterion C. Many candidates lost a mark due to improper notation and/or terminology or failing to define variables and teachers should take more care to point this out to their students. It is important that teachers write detailed comments on the front of the cover sheet explaining why the marks were awarded. They are also encouraged to make comments throughout the project in pencil in the margins and check the accuracy of the mathematics when assessing the work.

Candidate performance against each criterion

Criterion A

Many of the candidates were able to achieve level 2. Most projects contained a statement of task and a plan for carrying out the task. Very few candidates did not give their project a title. Candidates, generally, mentioned the mathematical processes they would use. However, often they did not justify the reason for choosing each of the processes carried out, thus depriving them of the highest award. Occasionally, processes not mentioned in the plan were carried out in the analysis; again, this deprived the candidates of achieving more than level 2. A little more work is needed on this criterion. Candidates need to be more aware that they should include all the processes used in the project in the introduction. Many candidates scored 2 out of the possible 3 marks here. This was mainly due to the fact that they did not give any reasons for the processes they were going to use.

Candidates with a clear statement of task and a detailed plan, which discussed the processes to be used and the rationale behind their choices, usually produced excellent projects.

Criterion B

Once again, most candidates were able to achieve level 2 since the data collected was often sufficient and organized/reorganized ready for analysis. Unfortunately the data collection process was not always clear. Candidates tended to say, “using a questionnaire” or “from the website”. They did not describe fully the sampling technique. Phrases such as “I chose at random fifty countries” were often seen. On the plus side when it was from a secondary source candidates generally gave the source of their data. The sampling process must be explained. If sampling is not done then this must be justified. More work is needed on this criterion with regard to sampling.

Many candidates collected data that was appropriate for their project but it was not always sufficient in quantity to perform the processes set out in their plan. If no real organization of the data is required then at most level 2 can be awarded for this criterion.

Raw data must be seen to consider level 2 for this criterion as tables of values and calculations must be able to be checked.

Data that is too simple also limits the marks for other criteria such as the mathematical processes, interpretation and communication.

Criterion C

It was good to see quite a few of the candidates using at least two simple mathematical processes along with a further process, usually either a χ^2 test or a scatter diagram and Pearson’s product–moment correlation coefficient. Occasionally candidates used both processes in their further analysis. In some schools candidates knew that they needed to apply Yates’s continuity correction to a 2 by 2 matrix; in other schools they did not. Often projects contained arithmetical errors or simple processes not relevant to the task. This meant that the candidate could not go beyond level 2. Frequently the mark awarded in criterion C was lower than the teacher awarded. Bar charts, pie charts and box-whisker plots were seen in the projects, as were percentages, the mean, median and quartiles. At times candidates showed too few calculations in these simple processes. Calculator generated results appeared without working or interpretation.

Most of the changes in the new assessment criteria are in this Criterion. Many more teachers and candidates appear to have paid attention to the changes this session than in May 2014.

To help teachers better assess criterion C, here follow some clarifications:

The candidates must complete at least two simple processes that are correct and relevant to be awarded level 3 for this criterion. It is required that only all simple processes are relevant at this level. Irrelevant further processes do not preclude the candidate being awarded level 3.

Simple processes are considered relevant if they pertain to the statement of task and if these processes are used later in the development of further processes, as stated in their plan.

If there are no simple processes in the project, then two of the further processes will be considered to be simple processes and **not** further processes.

Repeated processes count as one process (e.g. producing two bar charts).

If the project includes only two processes and one is incorrect, then level 1 is the maximum which can be awarded.

If there is only one process used, simple or further, then the candidate is awarded level zero. The only exception to this is if a χ^2 test is completed in full, by hand, and is the only process, then level 1 is awarded.

If the simple and further processes are not presented in order, the student will not be penalized in this criterion. However this may be penalized in criterion F.

To be awarded level 5 all further processes (and there only needs to be one) must be without error, and must be relevant.

Any process that is beyond the course needs to be fully explained to be considered a further process. For example the unsupported use of the *t*-test, whether performed wholly on the GDC or by substitution into the formula is deemed a simple process.

Although the processes are not limited to the χ^2 test and calculating the regression equation, the frequency with which they appear makes it worthwhile producing further guidance on how they should be marked.

χ^2 test

A χ^2 test performed by hand is considered to be one further process.

For a completed χ^2 test candidates are expected to write down their hypotheses, degrees of freedom, show how to calculate at least one expected value and complete the table of expected values, work out the χ^2 test statistic using the formula and write down the conclusion (using either the critical value or the significance level).

If the observed values are not frequencies, then at most level 3 can be awarded for criterion C.

If any expected values are less than 5, then at most level 4 can be awarded for criterion C, and only if all the working is shown in full. If the working is not shown, then at most level 3 can be awarded.

If the degree of freedom is 1, then Yates's continuity correction must be applied (and **only** when the degree of freedom is 1). If the correction factor is not applied and the test has been satisfactorily performed by hand then at most level 4 can be awarded.

Please note that a χ^2 test does not prove anything; it supplies evidence or support only.

Correlation / regression

If the candidate draws a scatter diagram and it is clear from the diagram that there is no correlation then it is relevant to calculate the correlation coefficient, *r*, to verify that fact. However, it is not relevant to calculate the regression line.

If from the scatter diagram it seems that there is some correlation then it is relevant to calculate the correlation coefficient, *r*, and, if the correlation is strong enough, then it is relevant to find the regression line, provided it is used or its purpose explained.

If a scatter graph is not drawn, then the relevancy of a regression line will depend on the value of *r*.

If the value of *r* is written down from the GDC (or Excel) then this is a simple process.

If the summary statistics have been calculated from the GDC and then substituted into a formula to determine *r* this is also a simple process.

Calculation of the mean or standard deviation as part of calculating r is not considered a separate process. The exception to this is if the mean or standard deviation has been calculated independently as part of the stated plan.

Normal distribution

Sketching a normal distribution curve and calculating probabilities or percentages is a simple process.

Using z-scores is also a simple process.

If a χ^2 goodness of fit test is performed by hand, then this is a further process.

Criterion D

The project flows better if the candidate writes partial interpretations/conclusions after each mathematical process. The stronger candidates had a detailed discussion of the results found. Overall this criterion was quite well addressed with many achieving level 2.

Most candidates managed to give at least one interpretation that was consistent with their analysis. However, the wording in this criterion has now changed and, if there are any inconsistent conclusions/interpretations, then there must be at least two consistent conclusions/interpretations for the candidate to be awarded level 2 marks.

Any irrelevant or unsupported conclusions (or personal beliefs) preclude the award of level 3.

Criterion E

Many candidates now show more understanding of validity and are able to comment meaningfully on the mathematical processes used or recognize limitations and provide a discussion.

Recognizing and commenting on the need to use the Yates's continuity correction factor or combining groups in the χ^2 test is sufficient for this criterion.

Despite what is written in some text books, all the expected values must be 5 or more and the only time that Yates's continuity correction factor is used is when the degree of freedom is 1.

Criterion F

Overall the structure of the projects was good. However, this criterion covers more than the layout now; it also deals with commitment. The project must demonstrate the required time commitment otherwise the maximum that can be awarded is level 1.

Some candidates included unsupported generalizations and this does not lead to a coherent project. Also, a large number of repetitive procedures preclude the award of level 3.

Graphs, tables or processes presented out of order also preclude the award of level 3.

If many pages of raw data or calculations via spreadsheet are presented, it is preferred that these be shown in an appendix; however this is not penalized.

If processes have been mentioned in the introduction and have not been performed or vice versa then the candidate is not penalized twice for the same error.

Criterion G

Surprisingly few candidates scored full marks on this criterion. The most common level awarded was 1 due to incorrect notation and/or terminology or failure to define variables.

Candidates that use Excel or calculator screen dumps need to be aware that this notation is not acceptable. If there are examples of such notation this must be explained and corrected in the body of the text.

Candidates should avoid using their cameras to take pictures of the calculator screens.

Isolated typographical errors are condoned, however if the candidate uses x^2 instead of x^2, for example, this is poor notation and the maximum that can be awarded is level 1.

Examples of notation:

Correct notation	**Incorrect notation**
x^2	$x \wedge 2$ **or** $x2$
$x \times 2$ **or** $2x$	$x * 2$
1.2×10^{-3}	1.2 E-03
χ^2	X^2 **or** x^2
r^2:Coefficient of determination	r^2:Correlation coefficient
$\sqrt{\frac{2402}{16}}$ **or** $\sqrt{}(2402/16)$	$\sqrt{}$ 2402/16 **or** sqrt.

Recommendations for the teaching of future candidates

- Read the Examiner's Report; this is extra important given the new set of criteria.
- Set internal deadlines for the project
- Have students assess previous projects to gain an understanding of the assessment criteria
- Encourage students to show hand calculations even if they are making use of technology such as Excel
- Help the students to understand how to address validity
- Encourage the students to use at least two simple processes in their analysis
- Make sure that the students define any variables in their project
- Show the students how to use equation editor and where to find the symbol for χ
- Show the students how to use Yates's continuity correction.
- Make sure that students attach all raw data
- Explain sampling to the students
- Show some/all calculations that lead to the results
- Fully explain the reasons for using the mathematical processes described in the plan

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 35	36 - 48	49 - 60	61 - 73	74 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Candidates struggled to find the area of a compound shape (two triangles)
- The candidates found it difficult to "justify" their answers and state what gradient and *y*-intercept of a linear function represents in context.
- The properties of a quadratic function were not well understood.
- Inverse normal distribution was not done well.
- Although in Paper 1 a correct answer earns full marks, in situations when their working needed to be checked (following and incorrect final answer) candidates did not present their working logically.

The areas of the programme and examination in which candidates appeared well prepared

- Candidates had a good understanding of descriptive statistics and χ^2 .
- Other strengths include Venn diagrams, substituting values into a given equation, exchange rates and differentiation of powers.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Students were able to find the maximum and interquartile range from a box and whisker plot as well as identify the median. However many did not realize that there are 25% of the values between the lower quartile and the median and that 75% of values are above the lower quartile.

Question 2

Most candidates were able to find the distance between two coordinates but only the best could find the area of a kite. This was attempted through a wide variety of methods e.g. use of the cosine rule. In reducing the kite to two triangles, some candidates used ABD and BCD as oppose to the much more helpful ABC and ACD.

Question 3

Students were able to identify if data were discrete or continuous, find the total frequency, mode and standard deviation using a calculator. A surprising number of candidates could not find the mean and confused this with the median or ignored the frequency (using frequency equal to 1).

Question 4

Currency exchange was well understood but frequently candidates rounded prematurely, truncated their answer or did not follow the directions to give the answer to two decimal places. Candidates should set their work out so that their path to the solution can be followed by the examiner.

Question 5

Many candidates confused the converse with the inverse. Some candidates were successful in correctly filling in the truth tables; others appeared to randomly write down T and F. Candidates need to provide a clear and explicit description of why two statements are logically equivalent i.e. state which columns in the truth table they are comparing to earn the reasoning mark. Candidates lost the final two marks for vague answers such as "they are the same".

Question 6

Many candidates forgot to subtract the principal value to find the interest earned. Lots of candidates did not round their answer to the required level of accuracy that was stated in the question.

Question 7

Few candidates were able to state what the parameters of a linear function represented in context. Those that set up their equations correctly were able to find the parameters correctly.

Question 8

A number of candidates did not know what a rhombus was. The concept of intersection was better understood than complement.

Question 9

Candidates were able to write down equations for the perimeter and area of a rectangle. However, from the numerical values of the perimeter and area they were unable to find the dimensions of the rectangle or left this part unanswered. Finding the area of a smaller rectangle as a percentage of a larger rectangle was well done.

Question 10

The χ^2 question was well answered although the weaker candidates confused independence and correlation as well as level of significance and critical value.

Question 11

Many candidates could find the *x*-intercept of a linear function, $f(x)$ and the *y* intercept of an exponential function, $g(x)$. However many gave the co-ordinates of the intersection of these two functions when asked to solve $f(x) = g(x)$. Only the very best candidates correctly gave the interval for which $f(x) > g(x)$.

Question 12

Finding the probability from a normal distribution was well done but only the stronger candidates could find the expected value and the inverse normal.

Question 13

Generally the exponential function was well understood. Many attempted to use logarithms rather than their calculators.

Question 14

Very few correctly drew the axis of symmetry (which was given) and drew a horizontal line rather than a vertical line. The quality of the parabolas drawn was poor. Properties of quadratics were not well understood. This is the question which candidates found most challenging.

Question 15

Most candidates were able to differentiate a quadratic correctly. Many used the original function rather than gradient to find the value of *a*.

Recommendations and guidance for the teaching of future candidates

- It is important that candidates write down the steps that have been used to find their answers; it is insufficient to write "used GDC". For example, the working that is expected to be seen when finding the mean is:

$$\frac{1\times 4+2\times 7+3\times 12+4\times 10+5\times 14+6\times 13}{60}$$

- Students found “justify” and interpretation of context challenging. Candidates should practise problems where the information is given descriptively rather than by an equation such as in the quadratics question.

Further comments

Some candidates did not know how to use their calculator for example to solve exponential equations. Teachers should explain how the GDC can be used to also find features of a graph and to do the statistical applications such as the normal distribution.

Mistakes in entering data into the calculator cannot be taken into account by the examiner so candidates should check carefully their entries to avoid a zero score on these types of questions.

Candidates should make sure they read the directions and give answers to the appropriate level of accuracy.

Candidates should be encouraged to show working, as follow through marks cannot be awarded without this.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 36	37 - 46	47 - 56	57 - 66	67 - 90

General comments

The majority of the candidates demonstrated good knowledge of the course material and ability to apply that knowledge to answer the exam questions.

The areas of the programme and examination which appeared difficult for the candidates

The following tasks proved to be challenging for the candidates: Calculating conditional probability, accurately drawing a regression line, coordinate geometry, calculus in context, using AP formulae in context, using the derivative of a given function for optimization, and using algebra and doing algebraic manipulations. Candidates had difficulty with the parts in questions 3, 4, and 6, which required “showing that” a statement is true. Many had problems with the geometrical properties of the curved surface area of a cone. Many candidates had difficulty interpreting the contextual questions. Some candidates lost all marks when they gave incorrect answers without showing their working.

The areas of the programme and examination in which candidates appeared well prepared

Good working was shown by the majority of the candidates so that follow through marks and method marks could be awarded where parts were incorrect. Many scripts were neatly presented although still not all candidates are organizing their working carefully. Drawing a scatter diagram, calculating and interpreting the Pearson's product-moment correlation coefficient, finding and using the equation of the regression line, calculating the percentage error, interpreting and using a tree diagram to calculate compound probabilities, the geometry of a circle and right angle triangle, gradient of a line and a line perpendicular, were well understood. The use of the sine and cosine rule to calculate the missing angle and side of a triangle, the formula of the area of a triangle, the formula for the *n*th term of an arithmetic sequence, the formula for the sum of *n* terms of an arithmetic series, were mostly successfully used. Very few candidates made an error rounding their answers. Premature rounding did not appear to be a major error in multi step problems. Students were proficient in using the features of their graphical display calculator. Those who did use the GDC showed working. Correct units of measurement were almost always shown. Most candidates were able to demonstrate good knowledge of the learned mathematical concepts and their applications.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: scatter diagram and regression line.

Most students took great care in drawing their scatter diagram. Axes were labelled and the given data points were plotted accurately. Most were able to use their GDC to correctly find the Pearson's product-moment correlation coefficient and the equation of the regression line with variable T. The air temperature was found by substituting 70 into their equation with no students using their graph to find this value. While most used a ruler to draw the regression line, many did not attempt this part or drew a curve through their points. In many cases the regression line did not pass through their mean point nor had the correct y-intercept. For the most part students substituted their correct values in the percentage error formula and got full marks for this part.

Question 2: probability

Most candidates answered part (a) well. Parts (b)(i) and (b)(iii) were also were done well but some students had problems with part (b)(ii). Most students were successful in comparing their results in part (c) and were able to correctly judge Sonya's statement. Some students based their conclusion on the total number of counted paths that lead to traps without critically evaluating Sonya’s statement. The responses to the part d) of the question were particularly weak, and many candidates were not able to recognize the conditional probability and calculate it. A common incorrect answer seen was 1/3.

Question 3: solid geometry & calculus

Very few candidates scored full marks on the "show that" question in part a). Many students did not equate the curved surface area of the cone to the area of a semicircle of 39.27 m^2. It was common to see students substitute the given slant height value into the surface area formula for the cone and use this to find the area of the base. Those who did use the formula for the area of the semicircle used the variable *r*, rather than *l*. While parts (b)(i) and (ii) were not always answered, in (b)(iii) most used Pythagoras' theorem to find the height of the cone correctly. In part (c) many students were not able to write the expression for the height of the cone explicitly. Also in part d) few students were successful in showing how the volume could be expressed as a function of *r*. Some students who managed to write the correct expression for *h* often missed the brackets when substituting it into the volume equation and thus lost the mark in this part. Finding the derivative in part (e) was well attempted by many candidates. While most of the candidates did well in this part, some had substituted π with 3.14 and lost one mark for insufficient accuracy. In part (f), many candidates who did equate their derivative to zero were successful in finding the value of *r* and the corresponding maximum volume. Some students, however, substituted the value of *r* in the derivative rather than the volume expression and lost the marks in (f)(ii).

Question 4: plane geometry & trigonometry

Part (a)(i) was well done on the whole and the cosine rule was successfully used to find the size of angle ACB. Although most candidates appeared to understand the congruency of angles ACB and DCE, many had difficulty communicating this. Some students incorrectly ascertained that AB and DE are parallel, and few candidates managed to give enough reasons to earn the last mark in part (a)(ii). In part (b)(i) many students had difficulty using algebra to find the size of angle DEC. In part (b)(ii) students usually substituted their appropriate values into the sine rule. The area formula was mostly used correctly in part (c) but not all candidates used the correct included angle. Some used alternative methods such as first finding the altitude to side DE, in order to find the length of DE. Correct units of measurement were usually included for all calculations.

Question 5: arithmetic sequences/series

Apparently many students did not read the question carefully and incorrectly interpreted what it means to collect a pumpkin. As a result, many students gave the half distances as answers in parts (a), (b)(i), (b)(ii), and (c). In part (a) a common answer was 3. Many candidates worked with half distances throughout the question, but managed to get follow through marks in the subsequent parts. Many students were able to substitute their values into the arithmetic series formula in part (d). As in past examination sessions, students had difficulty in part (e) with solving for *n*, the number of terms in an arithmetic series. Common errors included incorrect simplification of the quadratic expression, acceptance of a non-integer value of n, rounding their value of *n* up rather than down. The responses to part (f) were often weak. Students followed through with their value of *n* from part (d) into part (f) but few were successful in finding Peter's distance from the start.

Question 6: coordinate geometry & algebra

Parts (a), (b) and (c)(i) were in general correctly answered. Very few were able to gain full marks on the “show that” question in (c)(ii) – some gained the method mark for substituting their gradient in the equation of the line. Most were not able to choose the correct strategy in finding the equation of the line AB. Some candidates tried to use the given equation. Some found an un-simplified equation of the line AB, but were not able to algebraically manipulate it and simplify it to the given equation. In part (d)(i), some found the area of triangle OBC, but had difficulty using coordinate geometry to write and solve the non-numeric expression in part (d)(ii). Part (e) was poorly attempted and very few of those students who did it showed any working for finding the value of *a*.

Recommendations and guidance for the teaching of future candidates

Be prepared to work on contextual questions from different parts of the syllabus:

Students should read the exam questions carefully and note the information that is given in the question. Use diagrams and sketches to illustrate the information where possible. Candidates need to be encouraged to interpret questions, and to understand the concepts and reasoning underlying algorithms that are used. They should focus on uncovering the relevant maths in descriptions of real life situations.

Know the command terms:

Students should know all the command terms so that they know what action is required. They should understand the meaning of the term “show that,” the difference between “show that” and “verify,” and that the “show that” command requires students to state both the unrounded and the given rounded answer. They should also know the difference between “sketching a graph” and “drawing a graph,” and invest the appropriate effort in the given task. The command “draw” requires that graph be drawn accurately, a proper scale used, and axes labelled. A ruler should be used to draw a line.

Show working, and label the part of the question you are answering:

All relevant working should be shown in each question. Follow through marks can be awarded where appropriate. Proper labelling is necessary as much to help candidates quickly review their work at the end of the exam as for the examiners when they review and mark the work. Each new question should start on new page.

Use GDC more effectively:

Understand all the relevant functions and uses of the GDC. There is no need to explain how the GDC was used, i.e. which keys were pressed, etc. Candidates need to be encouraged to use their GDC throughout the entire course. Students should be aware of how to utilize the advanced features of the GDC. Care must be taken choosing an appropriate graphing window. Failure to do so may result in the student not identifying important features of the function. Familiarity in using the calculators to graph unfamiliar functions and using it to solve equations is essential**.**

Check answers carefully:

Candidates should be reminded to check their answers to ensure they are reasonable in the context of the question.

Pay attention to the required accuracy for specific answers

Candidates should be reminded to give their answers to the accuracy required by a question, or to 3 significant figures otherwise, and understand the marks that may be lost if they fail to do so. Premature rounding can be an issue for multi part questions, and students should be aware of it.

Learn to write succinct, clear, and well-grounded justifications:

It is important that students learn to communicate clearly. Teachers should demonstrate how to draw conclusions and write clear, succinct, and well-grounded justifications to support them. Students should be familiar with appropriate terminology for each area of the course, this is important when asked to explain their reasoning or give justifications.

Review past papers:

Candidates should familiarize themselves with previous papers, their format, and key terms that are used. It would be a good idea to ask students to work on questions for a given time so that they can learn how to manage their time during examination.